APPROXIMATE CALCULATION OF THE HIGH-FREQUENCY RESISTANCE MATRIX FOR MULTIPLE COUPLED LINES*

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Abstract

A new and relatively simple method for the calculation of the high-frequency resistance matrices of multiple coupled lines is presented. The method is based on a generalization of Wheeler's "incremental inductance rule" so as to apply to any number of coupled lines.

A. Introduction

This paper relates to the calculation of the high-frequency resistance matrices for multiple coupled lines. Such matrices are needed for applications such as the accurate analysis of microwave circuits which involve multiple coupled transmission lines and the computation of crosstalk between interconnects in high-speed digital circuits. A new and relatively simple means for computing high-frequency resistance matrices is presented. It can be shown that by adaptation of the "phenomenological loss equivalence method" of Lee and Itoh, our high-frequency resistance matrix results can be utilized to obtain approximate resistance matrices for any frequency. [1]

In a classic paper [2] Wheeler introduced what he termed as "the incremental inductance rule" (herein, IIR for short) which is a powerful tool for computing the high–frequency resistance of TEM–mode (or quasi–TEM–mode) single transmission lines.

In this paper Wheeler's IIR is generalized for use in computing the resistance matrices of multiple coupled lines.

B. The Incremental Inductance Rule (IIR)

Wheeler's IIR formula for the distributed resistance of a transmission line is (within a minus sign)

$$R = \frac{R_s}{\mu_0}\left(\frac{-\partial L}{\partial n}\right) \text{ohms / m} \qquad (1)$$

where R_s is the surface resistance $\sqrt{\pi f \mu \rho}$ of the conductor, μ_0 and μ are, respectively, the magnetic permeability in air and in the conductor, f is the frequency, and ρ is the resistivity of the conductor. Here the derivative $\partial L / \partial n$ is the incremental change in distributed inductance due to an incremental displacement of the conductor surfaces outward from the surface's initial location. (Wheeler defines the derivative for inward displacement hence does not have the negative sign.) It is interesting to compare the distributed resistance for the microstrip example in Fig. 1 as computed using the IIR, with the results computed using the program LINPAR in [3]. Here the IIR Eq. (1) was evaluated numerically using the methods of [4]. The IIR method yielded 9.25 ohms/m whereas LINPAR predicted 8.12 ohms/m. Which value is the most accurate is not certain. The methods of [4] have been found to give very accurate values for L and C so the numerical calculation of the derivative is believed to also be quite accurate. LINPAR in [3] uses a totally different approach which involves computing the power loss from the current distribution. The current distribution is represented by a series of pulse basis functions. These may not represent the current very well near the edges of the strip, especially for some of the examples below where LINPAR does not provide for the use of as many basis functions as one might like.

C. The Generalized IIR for Multiple Coupled Lines

An adequate explanation of the derivation of the generalized IIR for multiple coupled lines takes more space than is available in this publication, so herein we shall only present the results of the derivation, and a more complete paper will be presented later. The derivation yields for the elements of the resistance matrix of the coupled lines

$$R_{jj} = \frac{R_s}{\mu_0}\left(\frac{-\partial L_{jj}}{\partial n}\right)\Bigg|_{\substack{\text{conductors \#0} \\ \text{and \#}j\text{ perturbed}}} \qquad (2A)$$

$$R_{j,k}\Big|_{j\neq k} = \frac{R_s}{\mu_0}\left(\frac{-\partial L_{j,k}}{\partial n}\right)\Bigg|_{\substack{\text{conductor \#0} \\ \text{alone perturbed}}} . \qquad (2B)$$

In these equations conductor #0 refers to the "ground" or "reference" conductor. In microstrip conductor #0 is the ground plane, and in stripline it is the two ground planes. In many digital circuits the situation is more like that in Fig. 4 where any ground plane is very remote compared to the conductor spacings, and the voltages on lines #1, #2, and #3 are referenced to the "common return line" which, herein, is designated as line #0. As is seen from Eq. (2A), to compute, say, the self resistance R_{22} for cases such as those in Figs. 2 to 4 only the surfaces of conductors #2 and #0 are perturbed when computing the derivative, while the other conductor surfaces are unperturbed. When computing mutual resistances, such as R_{12}, only the common conductor (or conductors) #0 is perturbed. For example, when computing the mutual resistances

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numerically the derivatives are obtained by computing the inductance matrix for the unperturbed structure and then recomputing it with the surfaces of conductor #0 perturbed outward a small amount. From these results all of the derivatives needed for computing the mutual resistances are easily calculated.

Resistance matrices for a number of examples having two or more lines were computed using the IIR method above along with the methods in [4] for computing inductance values for the numerical evaluation of the derivatives. Corresponding calculations were also carried out using LINPAR [3]. All of the calculations were made for 10 GHz. For the example defined in Fig. 2 and its title we obtained

$$R = \begin{bmatrix} 2.49 & 0.37 \\ 0.37 & 3.87 \end{bmatrix}_{\mathrm{IIR}} \qquad R = \begin{bmatrix} 2.31 & 0.35 \\ 0.35 & 3.49 \end{bmatrix}_{\mathrm{LINPAR}} \tag{3}$$

For the example in Fig. 3 we obtained

$$R = \begin{bmatrix} 6.13 & 0.99 & 0.48 \\ 0.99 & 7.07 & 1.05 \\ 0.49 & 1.05 & 7.20 \end{bmatrix}_{\mathrm{IIR}} ,$$

$$R = \begin{bmatrix} 5.81 & 1.35 & 0.63 \\ 1.35 & 6.76 & 1.45 \\ 0.63 & 1.45 & 6.62 \end{bmatrix}_{\mathrm{LINPAR}} \tag{4}$$

while for the example in Fig. 4 we obtained

$$R = \begin{bmatrix} 74.08 & 29.03 & 28.03 \\ 29.23 & 65.87 & 26.46 \\ 28.26 & 26.48 & 63.85 \end{bmatrix}_{\mathrm{IIR}} . \tag{5}$$

LINPAR is not set up to treat the type of structure in Fig. 4 so we were not able to obtain a result for comparison in that case.

As can be seen from the foregoing examples the resistance–matrix results from the IIR method and LINPAR are reasonably well in agreement for purposes of practical engineering. As to which results are the more accurate is difficult to say at this point, though there appears to be basis for the possibility that the IIR results might be the most accurate. It is interesting to note that all of the mutual resistances in Eq. (5) are fairly close in value, lying between 29.23 and 26.46. This is because all of these mutual resistances are actually the distributed resistance of line #0, while the differences in the various values result from the somewhat different surrounding fields seen by conductor #0 under the various conditions for which the resistance is evaluated.

References

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[2] H. A. Wheeler, "Formulas for the Skin Effect," Proceedings of the IRE, vol. 30, pp. 412-424, September 1942.

[3] A. Djordjevic, R. F. Harrington, T. Sarkar, and M. Bazdar, Matrix Parameters for Multiconductor Transmission Lines: Software and User's Manual, Artech House. The software refers to this program as LINPAR.

[4] G. L. Matthaei, G. C. Chinn, C. H. Plott, and N. Dagli, "A Simplified Means for Computation of Interconnect Distributed Capacitances and Inductances," to be published in the IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems, April 1992.

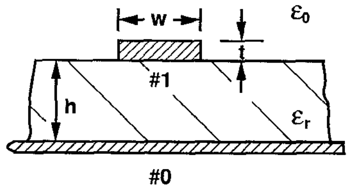

Figure 1 A single, copper microstrip example in which $w = h = 2$ mm, $t = 0.5$ mm, $\epsilon_r = 13$, and $\epsilon_0 = 1$.

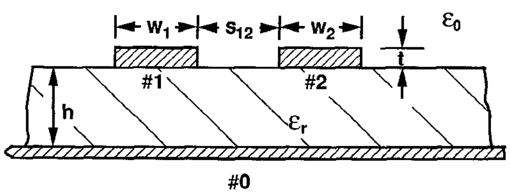

Figure 2 A two-line microstrip example in which $w_1 = 10$, $w_2 = 5$, $s_{12} = 5$, $h = 5$, and $t = 1$, all in mm, with $\epsilon_r = 13$ and $\epsilon_0 = 1$. The conductors are copper.

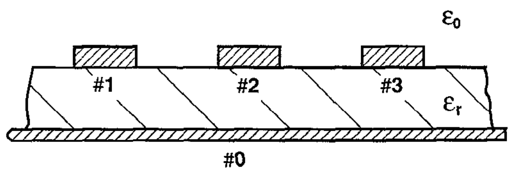

Figure 3 A three, copper microstrip example in which $w_1 = 4$, $w_2 = w_3 = 3$, $s_{12} = s_{23} = 2$, $t_1 = t_2 = t_3 = 1$, and $h = 2$ all defined in mm analogously to Fig. 2.

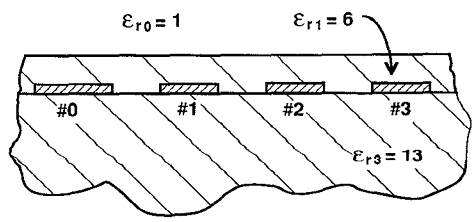

Figure 4 An example involving three, gold lines with a common return line. The dimensions in mm are $w_0 = 1$, $w_1 = w_2 = w_3 = 0.6$, $s_{01} = s_{12} = s_{23} = 0.50$, $t_0 = t_1 = t_2 = t_3 = 0.06$, and $h = 0.2$, all defined analogously to Fig. 2.